# Seismic diffractions: How it all began

Henning Hoeber[1*], Michael Pelissier[2], Tijmen Jan Moser[3] and Kamill Klem Musatov[4]


## Abstract

We review in historical order the key contributions to the development of the theory of diffractions. The work of Grimaldi, Huygens and Young provides the first part of this story, giving an understanding of diffraction and interference phenomena. Huygens was able to explain the laws of reflection and refraction, but lacked a deeper understanding of interference. This was provided by Young who used it to show how diffraction could arise from the interference of two waves. Fresnel, Helmholtz and Kirchhoff chose a different path and developed a full mathematical expression of Huygens' principle, incorporating wave phase and interference. Sommerfeld and his students were able to reformulate the Huygens-Helmholtz-Kirchhoff integral as the sum of an incident geometrical-optics wave and a diffraction integral, which is interpretable as the contribution of the diffracted rays from the boundary. From our modern vantage point, this provides a rather pleasing analogy to Young's early attempts at a theory of diffraction, using just two rays. A full ray-theoretical theory of diffraction, the Geometrical Theory of Diffraction, was given by Keller and extended by Klem-Musatov and Aizenberg to the case of seismic diffraction analysis.


## Introduction

Concepts such as Kirchhoff migration, the Fresnel zone and Young's modulus have long been known in the context of seismic reflection imaging. Less widely known is that Young, Fresnel and Kirchhoff were primarily pioneers in the theory of diffracted waves – the theory that deals with the waves generated by discontinuities in the propagating medium. This paper aims to fill that gap by reviewing the first principles of diffraction theory in order of its historical development.

## Grimaldi, Physicomathesis de lumine, coloribus et iride, Bononiae, 1665

The first written mention of diffraction is attributed to Francisco Maria Grimaldi (1618-1663), a Jesuit and professor of mathematics at Bologna. In his two-volume work, published posthumously in 1665, Grimaldi relates a set of experiments one of which is shown in Figure 1. A cone of light is passed through two small apertures (CD) and (GH). The outcome of the experiment, in contradiction to geometrical optics, is described by Grimaldi:

'When the light is incident on a smooth white surface it will show an illuminated base IK notably greater than the rays would make which are transmitted in straight lines through the two holes. This is proved as often as the experiment is tried by observing how great the base IK is in fact and deducing by calculation how great the base NO ought to be which is formed by the direct rays. Further it should not be omitted that the illuminated base IK appears in the middle suffused with pure light, and at either extremity its light is coloured.'

The section ends with the conclusion: 'It thus follows from the two experiments that light sometimes propagates in a peculiar mode and not in one of the three modes acknowledged by opticians, namely not in the direct mode, not in the reflected mode, and certainly not in the refracted mode, but in a fourth mode which has to be identified with a new name. We call this mode diffracted, since in this mode the light is diffracted and scattered into diverse luminous stripes.'

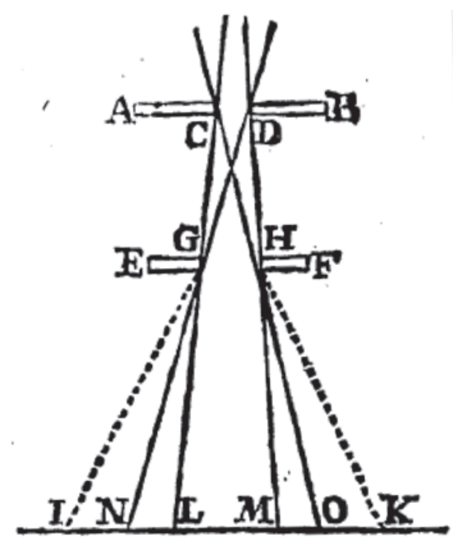



**Figure 1** Grimaldi's second experiment in De Lumine (1665).

## Huygens' Principle

At the same time as Newton was at work in England, Christiaan Huygens (1629-1695) in the Netherlands was developing a form of wave-theory, culminating in the publication of *Traité de la Lumière* in 1690. When a wave passes through matter each point on the wave front must be regarded as the source of a new

[1] CGG | [2] Integrated Geoscience Solutions | [3] Moser Geophysical Services | [4] IPGG SB RAS
[*] Corresponding author, E-mail: henning.hoeber@CGG.com

spherical wave with the same phase and one whose velocity depends on the medium.

Using Figure 2 Huygens explains how secondary waves combine to form the 'resultant' wave. Starting from point A, a wave propagates to form the wave front (surface) along HI. Consider a particle of the ether as stipulated by Huygens at one of the points denoted by *b* on this surface. It receives impulses from the particles immediately behind it and transmits impulses to the particles immediately ahead of it. Each point *b* is the source of a new wave. All of the secondary wavelets from the points *b* tend to cancel each other out except on the surface DF, to which the light propagates. Huygens manages to explain this without any use of the concept of interference when, in fact, this is exactly what he is describing. In his scheme the ether moves back and forth producing a pressure wave similar to sound in air. The sum total of all secondary waves then combines in such a way as to create zero amplitude except along the line of the wavefronts denoted by HI at an earlier time and by DF at a later time. Huygens finds that the wavefront passes instantaneously; there are no pre- or post-cursors.

Using his construction Huygens derived the laws of reflection and refraction. Huygens makes no reference to the phase of wave motion – which is why he is unable to explain interference and diffraction effects. Huygens postulates that the wavefront formed by the wavelets propagating in the opposite direction can be ignored. The modern formulation of Huygens' Principle incorporates this in form of the obliquity factor.

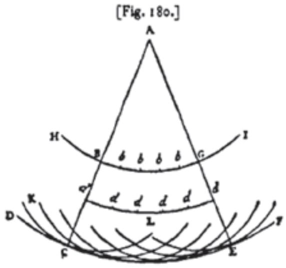
## Young: interference and diffraction

Starting in 1800 Thomas Young (1773-1829) published at least one treatise or lecture on vision, sound, light and colours every year for four years in a row and then summarized this work in 1807 in *A Course of Lectures on Natural Philosophy*.

On 2 November, 1803, Young presented at the Royal Society of London what is now called Young's experiment, in which he gave 'a proof of the general law of the interference of two portions of light'. It is perhaps less well known that Young did not, in fact, use the historic 'double-slit' experiment to show that light has wave-like behaviour and that it can interfere and be diffracted. Young's 1803 experiment was much simpler and 'may be repeated with great ease whenever the sun shines, and without any other apparatus than is at hand to every one'. The 'apparatus' used by Young was a 'slip of card, about one-thirtieth of an inch in breadth' which Young held edgewise into the sunbeam which was made to enter the room via a 'looking-glass', in other words, a mirror, and a small hole in a 'piece of thick paper' that Young had put in front of the window shutter that was also 'perforated with a fine needle'. If the beam of light has a diameter slightly larger than the thickness of the card then the beam is split into two. Young observed that 'the shadow itself was divided by similar parallel fringes' but in such a way as to leave the 'middle of the shadow always white' (all quotes from Young, 1804).

In 1802, Young read *An Account of Some Cases of the Production of Colours not hitherto described* at the Royal Society. In this paper, Young investigates two diffraction experiments: A slit formed by the edges of two knives and diffraction by a small body. In both cases, Young attempts to describe the diffraction pattern using just two waves, one of which is scattered at the boundary edge. This idea is a precursor to the modern geometrical theory of diffraction which we shall discuss in more detail below.

[Fig. 180.]




Il y a encore à conſiderer dans l'émanation de ces ondes, que chaque particule de la matiere, dans laquelle une onde s'etend, ne doit pas communiquer ſon mouvement ſeulement à la particule prochaine, qui eſt dans la ligne droite tireé du point lumineux; mais qu'elle en donne auſſi neceſſairement à toutes les autres qui la touchent, & qui s'oppoſent à ſon mouvement. De ſorte qu'il faut qu'autour de chaque particule il ſe faſſe une onde dont cette particule ſoit le centre. Ainſi ſi DCF [Fig. 180] eſt une onde emaneé du point lumineux A, qui eſt ſon centre; la particule B, une de celles qui ſont compriſes dans la ſphere DCF, aura fait ſon onde particuliere KCL, qui touchera l'onde DCF en C, au meſme moment que l'onde principale, emanée du point A, eſt parve|nuë en DCF; & il eſt clair qu'il n'y aura que l'endroit C de l'onde (p. 18). KCL qui touchera l'onde DCF, ſçavoir celuy qui eſt dans la droite menée par AB. De meſme les autres particules compriſes dans la ſphere DCF, comme *bb*, *dd* &c. auront fait chacune ſon onde. Mais chacune de ces ondes ne peut eſtre qu'infiniment foible compareé à l'onde DCF, à la compoſition de laquelle toutes les autres contri buent par la partie de leur ſurface qui eſt la plus éloignée du centre A.

**Figure 2** Figure from Huygens Traité de la Lumière (1690; reprinted in Crew, 1900).

## Fresnel and the synthesis of Huygens and Young

The key to Fresnel's work, culminating in his Mémoire Couronné of 1819, was the mathematical development of trigonometric wave analysis and the superposition of waves, as well as the adoption of Huygens' Principle.

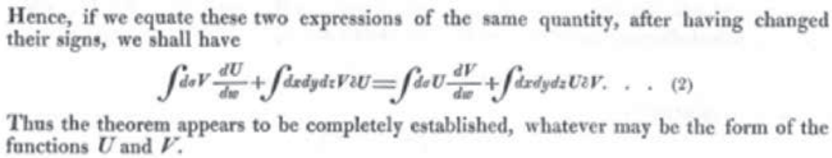
The first section of Fresnel's Mémoire is taken up with a detailed analysis of the failure of the theory of diffraction when using only two rays, as originally proposed by Young. Abandoning this approach, Fresnel then solves the problem of the interference of two harmonic waves with different amplitude, and a quarter wavelength phase-shift. He is able to show that the case of many interfering waves can be reduced to a similar form: 'Having determined the resultant of any number of trains of light-waves, I shall now show how by the aid of these interference formulae and by the principle of Huygens alone it is possible to explain, and even compute, all the phenomena of diffraction.'

In addition to Huygens' Principle (formulated in 43), Fresnel also states that '…I shall suppose that the velocities impressed upon the particles are all directed in the same sense, perpendicular to the surface of the sphere, and, besides, that they are proportional to the compression, and in such a way that the particles have no retrograde motion.'

The last constraint is the introduction of the obliquity (or inclination) factor. Fresnel later added a footnote in which he speculates that the inclination factor varies as the cosine to the normal (Buchwald, 1989).

Much is generally made on the response of Poisson, who, while investigating the memoir in detail, found that the Fresnel integrals could easily be calculated for the case of a circular diffracting body.

On performing the calculation, he found that the diffraction pattern predicted a bright spot at the centre of the shadow. This was immediately confirmed experimentally by Arago. However, it is not clear how much excitement this episode really brought about at the time (Kipnis, 1990) as 'Poisson's spot' was only mentioned briefly in Arago's report.



Hence, if we equate these two expressions of the same quantity, after having changed their signs, we shall have

$$\int d\sigma V \frac{dU}{dw} + \int dxdydz V\delta U = \int d\sigma U \frac{dV}{dw} + \int dxdydz U\delta V. \quad . \quad . \quad (2)$$

Thus the theorem appears to be completely established, whatever may be the form of the functions $U$ and $V$.

**Figure 3** Detail from Green's Essay showing the theorem that now bears his name.

## Green, Helmholtz, Kirchhoff, Rayleigh, Sommerfeld

The link between the theory developed by Huygens, Young and Fresnel and modern formulations of diffraction phenomena via the Helmholtz-Kirchhoff integral theorem is owing to George Green (1793-1841). Green sought to determine the electric potential within a vacuum bounded by conductors with specified potentials and he did so by first trying to solve the problem of a point source. He found that the potential could be determined by an integral over the boundary (Figure 3) if he knew the value of the potential of the boundary and the solution to the point source.

Green, of course, did not call his point-source solution the Green's function; this is due to Riemann (1826-1866). Carl Neumann (1832-1925) for the potential equation and then Helmholtz and Kirchhoff for the wave equation were soon to become the leading figures in the mathematical analysis of Green's functions. Kirchhoff derived the (retarded) Green's function for the three-dimensional wave equation:

$$G(r,t) = -\frac{1}{4\pi r}\delta(t-\frac{r}{c})$$

where $c$ is the propagation velocity. The delta function describes the wavefront spreading out in time, and is combined with a geometrical spreading factor. The solution is non-zero only at times $t=r/c$, which is a reflection of Huygens' Principle. The solution of the wave equation in two dimensions is very different, the wavefront does not just pass instantaneously, but remains excited for all times, although decreasing with time. Huygens' Principle in fact only works in odd dimensions other than one.

Using Green's theorem, Helmholtz (1859), and then Kirchhoff (*Zur Theorie der Lichtstrahlen*, 1882) derived integral formulations of the Huygens-Fresnel principle encapsulating that the wave disturbance at any one point is given by the superposition of secondary waves moving from the original excitation to the point of observation. Although in principle Kirchhoff's theorem offers an analytical solution to the boundary value problem, in order to determine the value of the scalar field we need to know its value and the value of its normal derivative over a surface surrounding the observation point. Not surprisingly then, further progress generally relies on assumptions or approximations to these quantities on the closed integration surface. While Kirchhoff obtained the field as a result of the interference of Huygens' secondary sources, Rayleigh (1842-1919) then used Kirchhoff's integral to show that the wavefield can be satisfactorily determined by the boundary conditions on the screen (1897). From the work of Sommerfeld (of whom more below) and Rayleigh (1897) it became clear that the study of diffraction of waves could formally be reduced to an ordinary boundary value problem of mathematical physics, in the form of integration of differential equations with partial derivatives, or to the solution of integral equations.

## Maggi-Rubinovicz and the rediscovery of Young's Boundary Wave

With Fresnel, Helmholtz and Kirchhoff we have a successful formulation of Huygens' Principle. In this theory, the boundary's role is to block the incident wave whereas secondary waves are emitted across the aperture.

Although Fresnel's synthesis of Huygens' Principle with Young's interference came to dominate the development of diffraction theory, we have also seen that the very first attempt to explain diffraction was given by Young (1802) using an alternative mechanism. He considered the phenomenon of diffraction to be the result of two waves – the direct, unobstructed portion of the primary wave (now called 'geometrical wave') from the source, existing only in the illuminated zone of propagation, and reflected waves (now called 'boundary diffraction waves') radiating from the edge of the screen in the illuminated region and in the shadow region. The superposition of these two types of waves gives the diffraction pattern as shown in a figure by Young (Figure 4).

In 1894 Sommerfeld obtained a rigorous and analytical solution to the problem of diffraction of plane waves by a planar, semi-infinite reflecting screen. The solution can be written as the sum of two terms. These are interpreted as a wave corresponding to the geometrical optics solution and a diffracted field from the boundary. In fact, it is a general feature of diffraction at an aperture that the total diffraction field is the sum of an incident wave and a boundary wave. The discovery and formulation of this result, based on Kirchhoff's work, is owing to Gian Antonio Maggi (1888), a student of Kirchhoff, Eugene Maey, a student of Sommerfeld and a little later to Adalbert Rubinowicz (1917, 1953), assistant to Sommerfeld. The boundary wave describes the superposition of spherical waves arising (scattered) at the boundary of the aperture. Discontinuities created by simple geometrical ray optics are softened by the use of full wave theory which compensates to

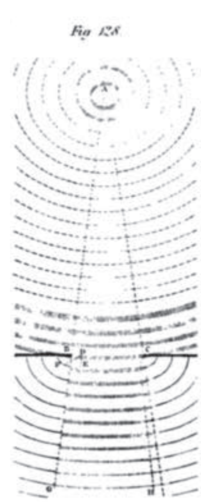



**Figure 4** Detail from Young's 1807 lecture showing the diffraction pattern created by the interference of the rays from the edge of the obstruction interfering with the rays passing unobstructed.

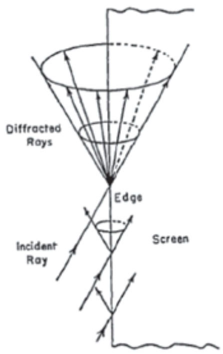



**Figure 5** Illustration of the cone of diffracted rays from a ray hitting the edge of thin screen obliquely (Keller, 1962).

create a continuous diffracted field everywhere. This extension to Kirchhoff's diffraction theory validated Young's attempt to explain diffraction as a deviation from the laws of geometric optics, which manifests itself in the form of diffracted waves. The modern formulation of this theory, known as 'geometrical theory of diffraction' is largely because of the seminal work of Keller. His landmark paper from 1953 on the representation of diffraction by rays is one of the most cited in modern diffraction literature. Keller (1962) notes that the theory was in part motivated by Sommerfeld's work on diffraction by a half plane (1896) as the cylindrical wavefronts for normal incidence on the edge lend themselves to a representation by rays. Generalizing Sommerfeld's results to oblique incidence the wavefronts become conical, so that a single ray incident on the edge forms a cone of diffracted rays. In analogy to reflected and refracted rays, Keller represents the diffraction energy generated by an incident ray by a diffracted ray (Figure 5). Diffraction by an edge is characterized by an edge diffracted ray, whereas diffraction from a vertex is characterized by a vertex diffracted ray. Keller generalizes Fermat's principle to formulate the law of edge, vertex and surface diffraction. The geometrical theory was extended by Klem-Musatov and Aizenberg (1984, 1985) to the seismic problem, and forms the cornerstone of the forward problem of seismic diffraction.

## Acknowledgements

This paper is an abridged and edited extract from the SEG reprint volume Classical and Modern Diffraction Theory edited by Klem-Musatov et al., 2016. The editors are grateful to the SEG for permission to publish this paper.

## References

Buchwald, J.Z. [1989]. *The Rise of the Wave Theory of Light.* The University of Chicago Press.

Crew, H. [1900]. *The Wave Theory of Light, Memoirs by Huygens, Young and Fresnel.* American Book Company.

Fresnel, M.A. [1819]. *Mémoire sur la Diffraction de la Lumière.* Translation by Crew, H. In: The Wave Theory of Light, Memoirs of Huygens, Young and Fresnel, American Book Company.

Grimaldi [1665]. *Physico-mathesis de lumine coloribus, et iride, Propositio 1.* Translation by Helbig, K., Jordan, W. and Schmoll, J. to be published in Hoeber et al., 2014] 1-11.

Helmholtz, H. [1859]. Theorie der Luftschwingungen in Röhren mit offenen Enden: Journal für die reine und angewandte Mathematik. *Crelle's Journal*, **57**, 1-72.

Huygens, C. *Traité de la Lumière, Chapitre 1, Des rayons directement étendus.* Reprint in: The Wave Theory of Light, Memoirs by Huygens, Young and Fresnel, American Book Company, 9-24.

Keller, J.B. [1962]. A geometrical theory of diffraction. *J. Opt. Soc. Am.*, **52**, 116-130.

Kirchhoff, G. [1882]. Berliner Sitzungsbericht. *Ann. der Physik,* **18**, 663.

Klem-Musatov, K.D., Hoeber, H.C., Moser, T.J. and Pelissier, M.A. [2016]. *Classical and Modern Diffraction Theory.* SEG Geophysics Reprint Series 29.

Klem-Musatov, K.D. [1994]. *Theory of Seismic Diffractions*. SEG Open File Publications No. 1.

Klem-Musatov, K.D., Aizenberg, A., Pajchel, J. and Helle, H.B. [2008]. *Edge and Tip Diffractions: Theory and Application in Seismic Prospecting.* SEG Monograph Series 14.

Klem-Musatov, K.D. and Aizenberg, A.M. [1984]. Ray method and the theory of edge waves. *Geophys. J. R. Astron. Soc.*, **79**, 35-50.

Klem-Musatov, K.D. and Aizenberg, A.M. [1985]. Seismic modeling by the methods of the theory of edge waves. *Journal of Geophysics*, **57**, 90-105.

Maggi, G.A. [1888]. Sulla propagazione libera e perturbata delle onde luminose in un mezzo isotropo. *Ann. Mat.*, **16**, 21–48.

Rayleigh, Lord (J.W. Strutt) [1897]. On the passage of waves through apertures in plane screens. *Philosophical Magazine*, 4**3**, 259-271, reprint in: Rayleigh's Scientific Papers, **4**, 283-296.

Rubinowicz, A. *Ann. d. Physik*, **53**, 257 [1917]; **73** [1924]; **81**, 153 [1926]; *Acta Phys. Polon.* **12**, 225 [1953].

Sommerfeld, A. [1894]. Zur Mathematischen Theorie der Beugungserscheinungen, *Gött. Nachr*, **4**. Translation by Nagem, R.J., Zampolli, M. and Sandri, G. in: Mathematical Theory of Diffraction. *Progress in Mathematical Physics*, **35**, Birkhäuser Boston.

Sommerfeld, A. [1896]. Mathematische Theorie der Diffraction. *Mathematische Annalen*, **47**, 317-374.

Young, T. [1804]. The Bakerian Lecture: Experiments and calculations relative to physical optics. *Philosophical Transactions of the Royal Society of London*, **94**, 1-16.

Young, T. and Wade, N. [2002]. *Thomas Young's Lectures on Natural Philosophy and the Mechanical Arts.* Thoemmes Continuum, Facsimile edition.

Doi: 10.3997/1365-2397.2017001