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FWI in Image Domain using Beam Tomography

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Summary

Beam Tomography (BT) is a super-efficient, high-resolution, wide azimuth tomography algorithm that can be formulated as Full Waveform Inversion (FWI) in image space, and is two to three orders of magnitude faster than the standard FWI in data space. The two methods can be used together to speed up the convergence of the FWI in data space, or separately, depending on the imaging objectives. We present details of the implementation of the Beam Tomography based on the Fast Beam Migration (FBM) algorithm and show results on synthetic and real data, and comparisons with standard FWI in data space. The Beam Tomography uses Fast Beam Migration to directly output at each (x,y,z) analysis point to 5,000-10,000 values of independent residual velocity measurements, a large increases over the 50-300 values used in standard tomography, providing a higher resolution velocity update. In addition, the Beam Tomography output contains image point azimuth information and this allows the tomographic update to go beyond the current limitation of limited surface azimuth velocity updates.

Theory / Method / Workflow

We begin with a short review of traditional tomography and Fast Beam Migration, since they are at the core of the velocity-building method that we propose. Next, we explain how components of traditional tomography and Fast Beam Migration can be put together to make a fast velocity building tool. We compare the standard FWI theory implementation in data space with the FWI in image space. Finally, we show examples of the velocity-building method in practice.

Standard Tomography

Reflection Tomography is an iterative inversion method that updates the velocity model and minimizes the deviation in the Common Image Gathers (CIG) from a flat event. In our implementation, we select special image points based on dip and event coherency called back-projection points (BP), from which the tomography traces rays back to the surface in order to distribute the velocity residual values throughout the velocity model. Rays from different back-projection points illuminate parts of the overburden (see Figure 1), and an appropriate compromise between velocity residuals coming from different rays is made by solving a least-squares problem.

After the seismic data has been migrated using the current interval velocity model, consistency of the model with the data is assessed by examination of the move-out in the Common Image Gathers, representing variation over different wave paths in the predicted depth of subsurface reflection events.

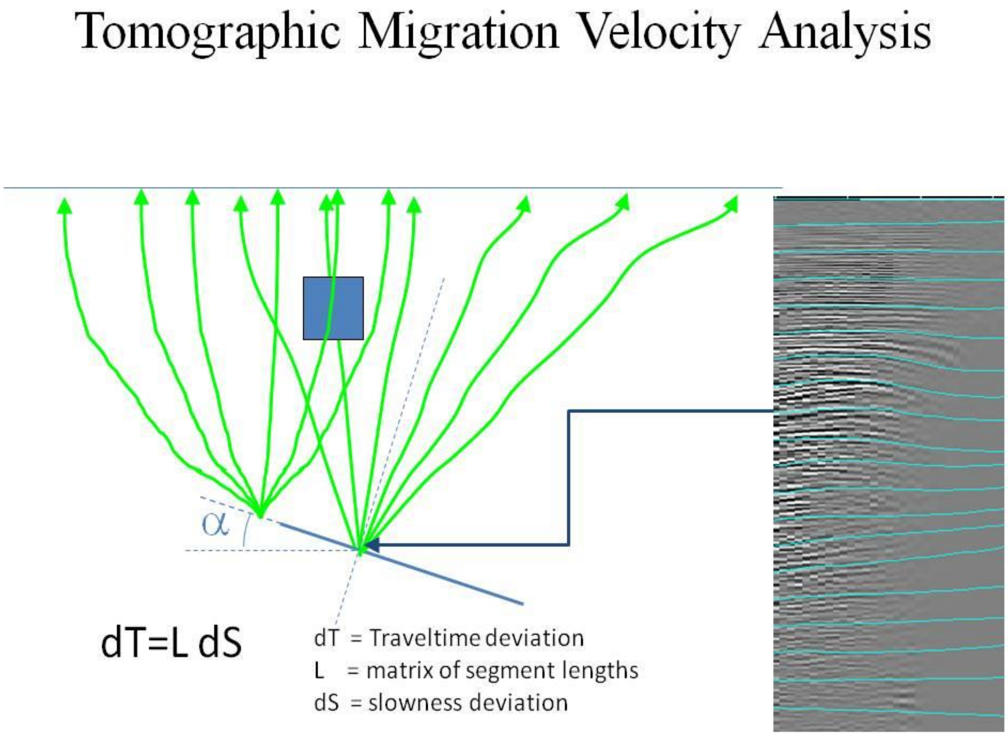

Figure 1: *Traditional tomography velocity update. Each MVA image point is used to trace a fan of rays to the surface. Information from the image gathers is then combined with the ray paths to update the velocity*

In the single-parameter update, the move-out is quantified through Semblance Analysis, which associates with each image point one or more velocity residual values, indicating whether the velocity in parts of the overburden visible to that image point is too high or too low. In the multiple-parameter update, the move-out or time-delays for a given event and all the offsets associated with that event in a CIG, is quantified through curve-fitting, using Plane Wave Destructor (PWD) filters to evaluate the dip and move-out of specific back-projection points from the stack.

The reflection tomography performed in the post-migrated domain has many advantages over standard tomography performed on prestack data (Stork, 1992). In general, post-migrated events are much easier to pick, the data volume is more manageable, and the whole process is more robust. The procedure converts common image gather residual picks to velocity changes using 3D tomographic back-projection. In tomographic MVA, fans of rays with the correct wave propagation geometry are used to back-project residual velocities to the places where the velocities errors originated. The extent of deviation at every offset corresponds to a residual traveltime associated with a ray pair from a source point to an image point and back to a receiver point. Tubes of ray pairs from an analysis image point illuminate part of the overburden velocity, and several overlapping ray tubes can be used to reconstruct the overburden velocity properties in a tomographic way. The resolving power of the tomographic method derives from illumination of velocity model cells under different angles with rays from different image points. In the tomographic reconstruction part, ray paths, computed residual traveltimes, and the unknown residual slowness field comprise a linear optimization system, which is solved by the method of conjugate gradients.

Fast Velocity Iterations

One key component of the fast velocity iteration workflow is the fact that the Fast Beam Migration allows the output of auxiliary velocity update information. We note that in traditional tomography, the gathers are scanned for an appropriate velocity perturbation at each analysis point and ray tracing is used to correctly spread this velocity perturbation into the velocity model. Since FBM is a ray based method, the information need by tomography to spread the velocity perturbation is readily available. Furthermore, since beams represent coherent events, the necessary velocity perturbation can be computed by comparing individual beams to the stacked beam image. Combining these two pieces allows us to replace the time consuming sequence of generating image gathers, post-processing and conditioning them, scanning them for single parameter (hyperbolic moveout) or multi-parameter residual curvature (non-hyperbolic moveout), picking residual semblance and QC-ing, and ray-tracing. Thus, the auxiliary velocity update information from FBM can be immediately used as input for the velocity update stage of tomography, greatly reducing the velocity update iteration time.

Short Review of Gaussian Beam Raytracing and Tomography

We define a Gaussian beam (Fomel and Tanushev, 2009) as a seismic event characterized by a particular arrival time, location, amplitude, orientation, curvature, and extent. The extent of a beam is controlled by an amplitude taper, which can be understood as the imaginary part of a complex-valued event curvature. In the process of seismic imaging, the beam changes its position in time and space, as well as its amplitude, orientation, and complex curvature. Neglecting higher-order effects, a Gaussian beam representation is a powerful asymptotic approximation for describing different wave propagation phenomena (Popov, 1982; Babich and Popov, 1990; Bleistein and Gray, 2007; Kravtsov and Berczynski, 2007).

Fast Beam Migration (FBM) is a fast method for producing seismic images. It takes the recorded seismic data and a velocity model and produces an image of the subsurface. A typical beam migration workflow contains the following steps:

1. *Beam Forming* – The seismic input data is analyzed for locally coherent events. The slope of these events is identified and the associated wavelet is recorded as a beam. Beams are multidimensional objects that contain the recording time, the position of the source and receiver, the incident wave angles at the source and the receiver, and the associated seismic wavelet. This step needs to be done only once since it is independent of velocity.
2. *Beam Propagation* – This stage finds the migration time for each beam using ray tracing. For each beam, two rays are traced – one from the source and one from the receiver using the slopes identified in the beam forming stage. The time at which the rays meet in the subsurface is the migration time. All of the beam parameters are propagated to this time. These parameters provide information on how to reconstruct the wave field in the subsurface to form the image.
3. *Image Forming* – The final stage is to form the seismic image using the propagated parameters from beam propagation. At this stage, we can output an offset gather seismic volume that can be used as the input for traditional tomography or a stacked image. We

note that due to the parsimonious nature of fast beam, the raw stack is computed quickly and does not usually require any post-processing in order to be used for the analysis outlined below.

Migration Velocity Analysis (MVA) improves the velocity model associated with a survey in order to create an accurate image of subsurface structures. This is done by carefully analyzing the data and exploiting the fact that subsurface reflectors are evident in the data at different source and receiver configurations. Typically, traditional tomography takes as input a seismic image volume comprised of image gathers. This volume contains many images of the subsurface, each resulting from different pairs of sources and receivers (offset); however, tomography can use images separated by an attribute other than offset as well, such as angle or p value. A typical workflow for tomography includes the following steps:

1. *Data Preparation* – The gathers generated from migration are cleaned and preprocessed to facilitate better semblance analysis.
2. *Generating Picks* – points are picked manually or automatically in the subsurface of the earth that will be used for velocity analysis.
3. *Semblance Analysis* – the seismic gathers are analyzed at the generated picks to quantify the mismatch between subsurface images, and measure time delays along rays due to velocity errors in the model.
4. *Ray Tracing* – fans of rays are traced back to the surface from each of the subsurface picks.
5. *Velocity Update* – the velocity is updated along each of the rays with certain constrains (since many rays can pass through the same velocity sample) so that the subsurface images are better focused when imaged with the new velocity.

3D Residual Beam Traveltime

Examining the algorithm used in the Beam Propagation stage of FBM, we note that a small increase or decrease in the traveltime along the source and receiver rays will shift the beam imaging location in the direction normal to the reflector that the beam is imaging. Thinking of this procedure in reverse, we can determine a residual traveltime shift along the rays that will align the beam with the reflector that it is imaging. This, of course, is well known and used in traditional tomography during the semblance scanning of the gathers. However, in beam migration, this analysis can be carried out in a much more efficient manner: Each migrated beam represents a localized portion of the seismic image with limited extent and can be easily compared and synchronized to the stacked image, which is produced by all beams. This is simply done by cross-correlating the beam with the stack.

We emphasize that the alignment procedure is done with a stack and image gathers (common image, angle, etc) are not required. Furthermore, we note that since the beam can arrive at the reflector at any azimuth and reflection angle, the information obtained with this method is truly 3D and along with the beam source and receiver rays is all of the information required to update the velocity in the last step of traditional tomography. The procedure for determining the traveltime residual shift in the context of beams was first introduced by Sherwood, et al (2014), where the authors refer to it as “3DRMO”.

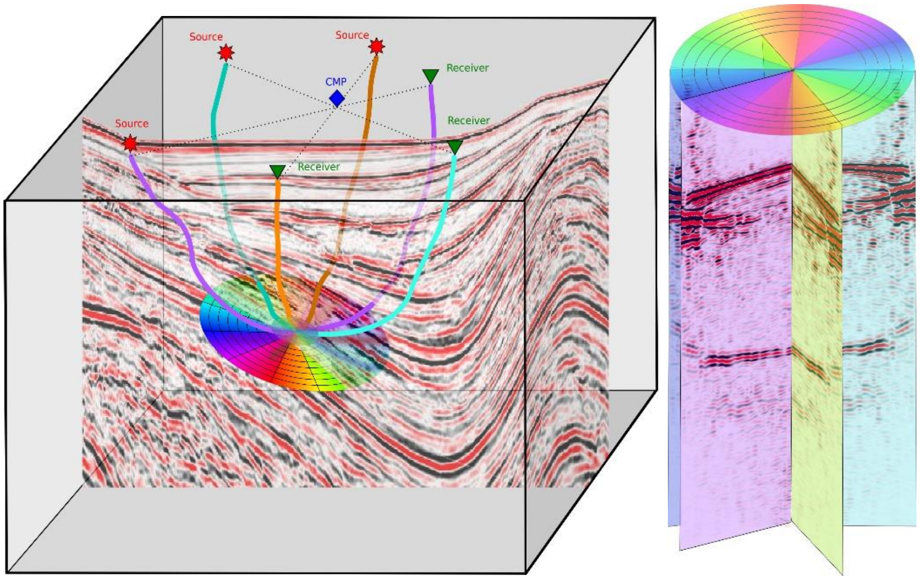

Figure 2: *Pictorial representation of the information contained in 3DRMO. The image at every point is obtained from summing the contributions of many beams. Each of these beams has its own unique combination of offset and azimuth (or equivalently reflection angle and subsurface azimuth). Thus the RMO values that flatten the gather depend on offset and azimuth.*

Beam Tomography

By combining Fast Beam Migration and reflection tomography, we can eliminate the first four steps of reflection tomography and replace them with an automated 3D residual beam traveltime shift calculation. Since the beam forming step of beam migration needs to be done only once for a given seismic data set, the entire iterative procedure of velocity building using beam tomography is reduced to: beam propagation, residual calculation and velocity update. These steps require no input from the user and can be iterated several times before the user QCs the results. This significantly shortens the computation time between successive velocity updates. In addition, there are other significant time savings as the traditional imaging workflow needs to preprocess the image gathers before tomography. One of the features of FBM that sets it apart from other types of migration such as reverse time migration or wave equation migration is that beam migration contains direct information about the connection between events in the seismic image and events in the seismic data.

During the beam propagation stage of FBM, for each beam we trace a source and receiver ray that meet at the reflector that the beam is imaging. If the image produced by a particular beam is not in agreement with the images produced by other beams of this reflector, we need to update the velocity. The only part of the velocity that will affect the beam is concentrated near the beam rays. Thus, for each beam, we form a row of the tomography matrix L that contains the paths of the source and receiver rays. We use the 3DRMO time shift for each beam to form the right-hand side $\Delta\tau$ of the tomography equation (1). A typical beam tomography work-flow contains the following steps:

- **Beam Propagation:** This stage is identical to the ``Beam Propagation'' step in FBM, with the additional output of the tomography matrix, which is formed using the source and receiver rays. Each source and receiver ray pair generate a matrix row.
- **Image Forming:** Using the propagated beam we form a raw stacked volume. In general the stack is small and formed much faster than gathers simply because the size of the data is much smaller.
- **3D RMO Computation:** Using the stacked image, we locally shift the beams at their imaging location so that they better align with the stack. This relative residual is used to produce the right hand side of the velocity update equation.
- **Adjacency Matrix Computation:** For each beam, we determine which other beams were used to produce the stacked image with which the beam was aligned in the ``3D RMO'' step.
- **Velocity Update:** This step is identical to the ``Velocity Update'' stage in traditional tomography.

This flow can be automatically iterated to produce velocity updates, since there are no steps that require user intervention. The user can QC the velocity and the raw stack for each iteration. The beams produced in the ``Beam Propagation'' stage can be used to from image gather for some of the iteration to provide additional QC. An example of a velocity built using automatically iterated beam tomography is shown in Figure 3.

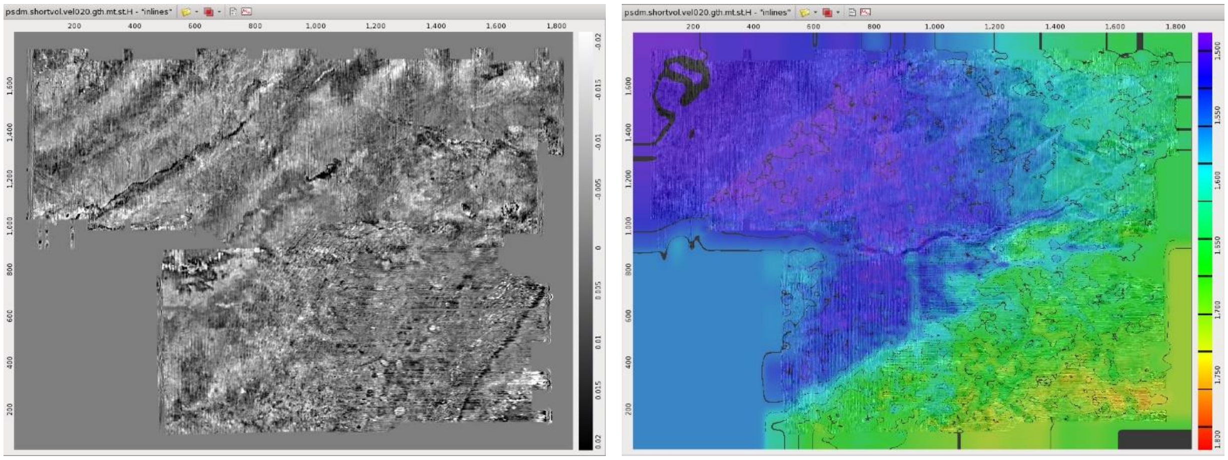

Figure 3: *Left: Constant depth slice though the migrated stack. Right: The Beam Tomography velocity overlaid with the stack. Notice the channel and the velocity inside the channel for an example of the high resolution achieved with this method. The updated velocity was obtained through 80 automatic iterations of beam tomography.*

FWI in Data Space and FWI in Image Space

FWI in data space inverts for the velocity model by solving a nonlinear inverse problem minimizing the difference between modeled data (MD) and recorded field data (Tarantola, 1984).

MD = F m, min ||Observed Data – MD||

The model is updated iteratively using a relation: $m_{k+1}=m_k+a\ \Delta m_k$ where Δm_k is a change of the model that minimizes the mis-match between the simulated and observed wavefields. The objective function (OF) measures the match between simulated and recorded data.

FWI in image space applies the inverse migration operator to the Tarantola FWI formulation to obtain the same equation in image space.

$m = F^{-1}$ OD, min $||m_i - m_{i-1}||$

The model is updated iteratively until the differences between two models are smaller than an epsilon. Diaz, Sava and Yang (2013) use the same minimization scheme but employ extended gathers. Beam Tomography uses hundreds of iterations of tomography and Fast Beam Migration to update the model.

Conclusions

We present a high resolution, wide azimuth velocity model building algorithm based on the Fast Beam Migration algorithm. Beam Tomography allows for faster turnaround time for large 3-D seismic projects and at the same time increases the accuracy of the velocity model by using wide azimuth information for tomographic updates.

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